Some econometric results for the
Blanchard-Watson bubble model

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The purpose of the present paper is to analyse a simple bubble model suggested by Blanchard and Watson. The model is defined by

\[ y_t = s_t \rho y_{t-1} + \varepsilon_t, \quad t = 1, \ldots, n, \]

where \( s_t \) is an i.i.d. binary variable with \( p = P(s_t = 1) \), independent of \( \varepsilon_t \) i.i.d. with mean zero and finite variance. We take \( p > 1 \) so the process is explosive for a period and collapses when \( s_t = 0 \). We apply the drift criterion for non-linear time series to show that the process is geometrically ergodic when \( p < 1 \), because of the recurrent collapse. It has a finite mean if \( p \rho < 1 \), and a finite variance if \( p \rho^2 < 1 \). The question we discuss is whether a bubble model with infinite variance can create the long swings, or persistence, which are observed in many macro variables. We say that a variable is persistent if its autoregressive coefficient \( \hat{\rho}_n \) of \( y_t \) on \( y_{t-1} \), is close to one. We show that \( \hat{\rho}_n \overset{P}{\to} p \rho \) if the variance is finite, but if the variance of \( y_t \) is infinite, we prove the curious result that \( \hat{\rho}_n \overset{P}{\to} \rho^{-1} \). The proof applies the notion of a tail index of sums of positive random variables with infinite variance to find the order of magnitude of \( \sum_{t=1}^n y_{t-1}^2 \) and \( \sum_{t=1}^n y_t y_{t-1} \) and hence the limit of \( \hat{\rho}_n \).